# KUANTUM BİLİŞİM

## (Bilim ve Aydınlanma
## Aykut Göker ile birlikte)

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**17 Kasım 2017, Saat 16:00**
**ANKARA ÜNİVERSİTESİ SİYASAL BİLGİLER FAKÜLTESİ**
**21. Yüzyıl İçin Planlama Seminerleri**

# İçerik

1. **KUANTUM BİLİŞİM KURAMI**
2. **KLASİK BİLGİ ve KUANTUM BİLGİ**
3. **KUANTUM MANTIK GEÇİTLERİ ve DOLANIKLIK**
4. **KUANTUM UZ-AKTARIM (*Teleportation*)**

***İlk kuantum mekaniği derslerimi aldığım değerli hocam***

# ***Burhan Cahit Ünal'a,***

***en derin şükranlarımla…***

# Bilişim Bilimi (*Information Science*):

Bir **sistemin (fiziksel) durumlarıyla** taşıyabileceği bilgilerin; sisteme kodlanması (***encoding***), işlenmesi (***processing***), iletilmesi (***transmission***), depolanması (***storage***), ``okunması'' (***decoding***) ve bu amaçlar doğrultusunda sistem tasarım ve üretimi ile ilgili her türlü etkinlik.

# Fiziksel Sistemler ve Fiziksel Durumları

1. Klasik Mekaniksel Sistemler (makroskopik sistemler)

$$(\mathbf{q}, \mathbf{p}) = (q^1, \dots, q^n; p_1, \dots, p_n)$$

2. Kuantum Mekaniksel Sistemler (atom, molekül sistemleri)

Bir saf kuantum durum $|\psi(t)\rangle \in \mathcal{H}$ Hilbert uzayı

$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathrm{H}|\psi\rangle$$

(Schrödinger denklemi)

Erwin Schrödinger (1887-1961)

David Hilbert (1862-1943)

$\psi(\mathbf{q}, t) = \langle \mathbf{q} | \psi \rangle$ $\longleftrightarrow$ Konum olasılık genlikleri

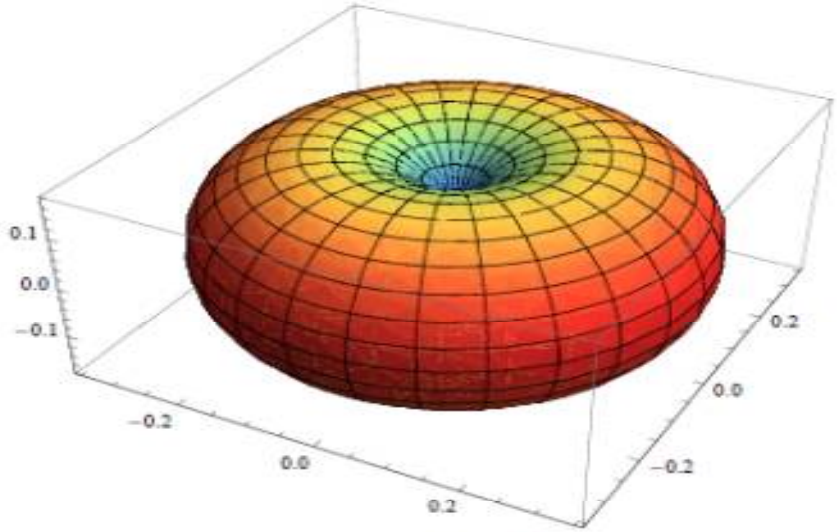
$\mathrm{P}(\mathbf{q}, t) = |\langle \mathbf{q} | \psi \rangle|^2$ $\longleftrightarrow$ Konum olasılık yoğunlukları

$\mathbf{q} = (\mathrm{x}, \mathrm{y}, \mathrm{z})$

$< \psi | \psi > = \int |\langle \mathbf{q} | \psi \rangle|^2 \, d^3\mathbf{q} = 1$ $\longleftrightarrow$ Boylandırma (normalizasyon)

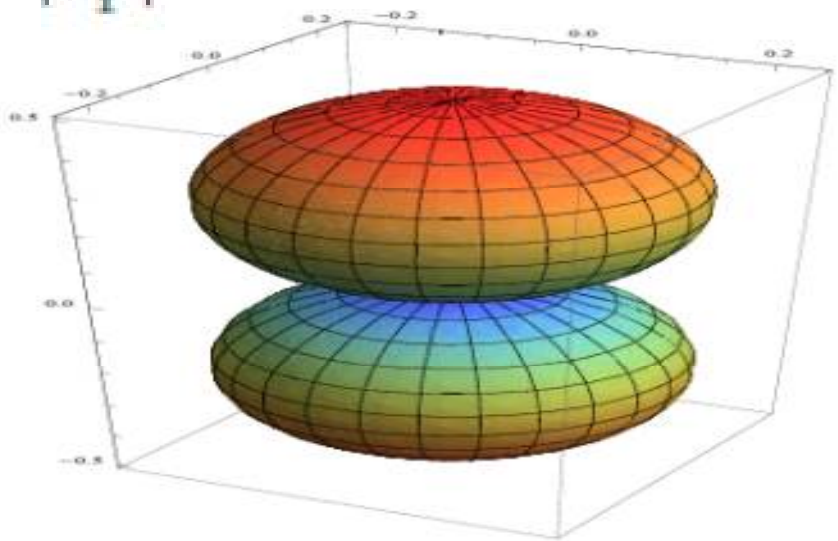
$\int |\mathbf{q} >< \mathbf{q}| d^3\mathbf{q} = \hat{\mathrm{I}}$ $\longleftrightarrow$ Konum tamlık bağıntısı

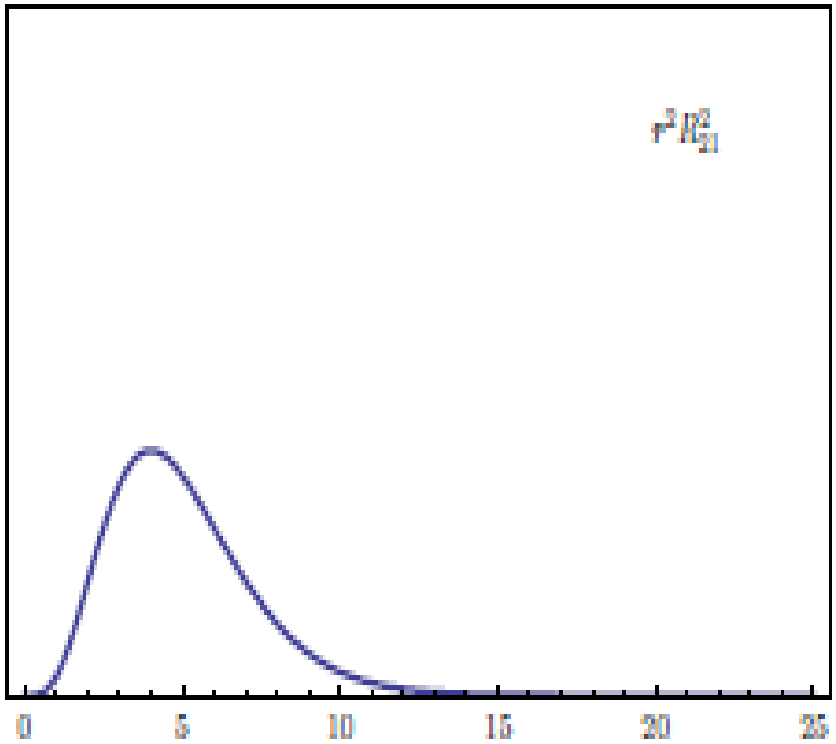
$< \mathbf{q} | \mathbf{q}' > = \delta^3(\mathbf{q} - \mathbf{q}')$ $\longleftrightarrow$ Diklik (ayırdedilebilme)




(a) $|Y_1^1|, |Y_1^{-1}|$

$|Y_1^0|$



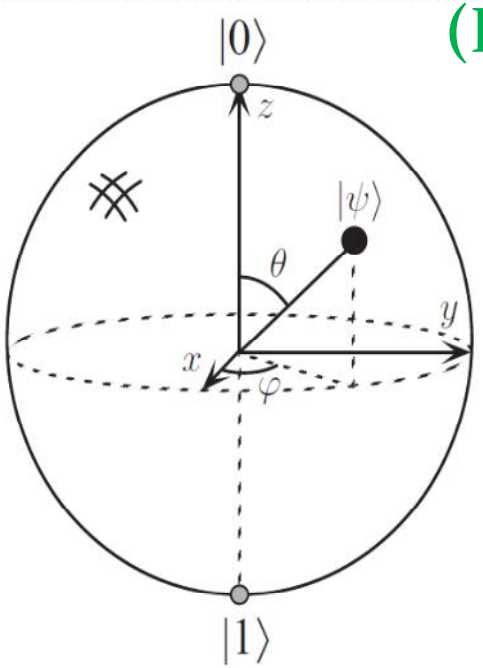


# Kübit sistemleri: $\mathcal{H} \cong \mathbb{C}^2$

1. ***Spini*** $\frac{1}{2}$ ***olan herhangi bir parçacık***
2. ***İki kutuplanma durumuyla bir foton***
3. ***İki enerji düzeyli herhangi bir atom veya molekül***
4. ***Süperiletken kübitler***
5. ***İki kollu MZ − girişimölçeri***
6. ***Katı − hal kübitleri***

$qubit = quantum\ bit$

**(B. Schumacher, 1995)**




Tek **kübit** için bir baz

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

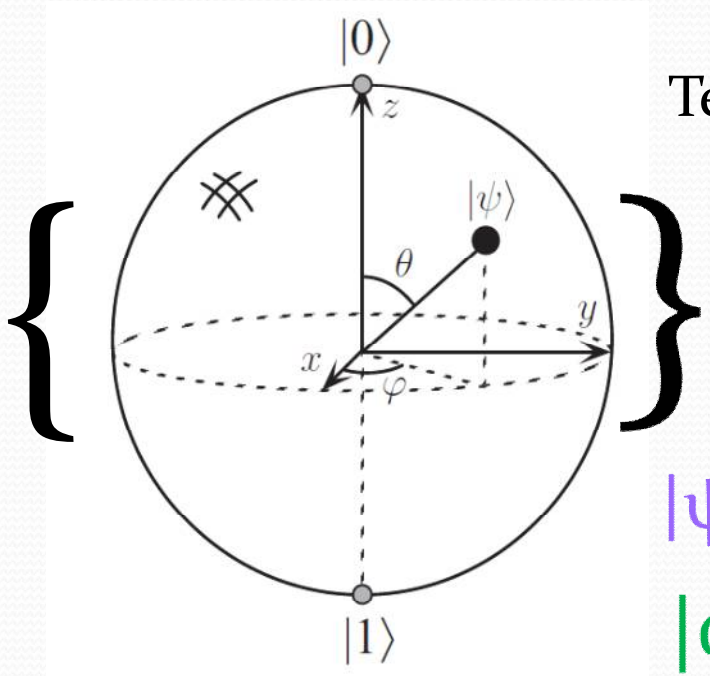
**(Stokes-Poincaré-Bloch yuvarı)**

# Bite karşılık kübit

$bit = binary\ digit$

$\mathcal{H} \cong \mathbb{C}^2$

$\{0, 1\} \leftrightarrow$

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$$|\alpha|^2 + |\beta|^2 = 1$$

**(Stokes-Poincaré-Bloch yuvarı)**

TABLO I: Bugünkü doğa ve evren anlayışımızı şekillendiren iki hareket kuramı olan, klasik mekaniğin ve kuantum mekaniğinin bir nitel karşılaştırılması.

| | KLASİK MEKANİK | KUANTUM MEKANİĞİ |
|---|---|---|
| ANLAYIŞ | İndirgemeci | Bütünlükçü |
| ÖNGÖRÜ | Belirlenimci | Olasılıkçı |
| EVRİLME | Çizgisel değil | Çizgisel ve Üniter |
| GENİŞLEME | Kartezyen | Tensörel |
| BİLEŞİM KURALLARI | Sıra-değişimli | Sıra-değişimsiz |
| ÖLÇÜM | Tersinir | Tersinmez |
| GERÇEKLİK | Nesnel | Öznel |

TABLE II: A qualitative comparison of two theories of motion, classical mechanics and quantum mechanics that shape our current understanding of the nature and universe.

| | CLASSICAL MECHANICS | QUANTUM MECHANICS |
|---|---|---|
| UNDERSTANDING | Reductionist | Holistic |
| PREDICTION | Deterministic | Probablistic |
| EVOLUTION | Nonlinear | Linear and Unitary |
| ENLARGEMENT | Cartesian | Tensorial |
| COMPOSITION RULES | Commutative | Non-commutative |
| MEASUREMENT | Reversible | Irreversible |
| REALITY | Objective | Subjective |

Tablo 3: **Saf Kuantum Durumlar.** Kuantum mekaniğinin formülasyonunda kullanlan temel matematiksel kavram ve yapıların (sağ sütun) uygulamadaki işlevsel anlamlarını (sol sütun) gösteren özet tablo.

| | |
|---|---|
| Bir sistemin olası tüm kuantum durumlarının uzayı | Bir $\mathscr{H}$ Hilbert uzayı |
| Bir kuantum durum | $\lvert\psi\rangle \in \mathscr{H},\ \lvert\psi\rangle \neq 0$<br>Bir kompleks vektör |
| Yalıtılmış kuantum sistemlerinin dinamiği | Schrödinger denklemi<br>$i\hbar\frac{\partial}{\partial t}\lvert\psi\rangle = \hat{H}\lvert\psi\rangle$ |
| Yalıtılmış sistemlerin zamanla gelişiminin üniterliği | $\hat{U}^\dagger\hat{U} = \hat{I} = \hat{U}\hat{U}^\dagger$<br>$\lvert\psi(t)\rangle = \hat{U}(t,t_0)\lvert\psi(t_0)\rangle,\ t \geq t_0$ |
| Koherent üstüste gelim | $\lvert\psi\rangle = \sum_i c_i\lvert\psi_i\rangle,\ c_i \in \mathbb{C}$<br>süperpoze durumlar |
| Tam ayırt edilebilir durumlar | Dik vektörler<br>$\langle\phi\lvert\psi\rangle = 0,\ \lvert\psi\rangle \neq 0, \lvert\phi\rangle \neq 0$ |
| Aynı kuantum durumlar | $\lvert\langle\phi\lvert\psi\rangle\rvert = 1 \Leftrightarrow \lvert\psi\rangle = e^{i\alpha}\lvert\phi\rangle, \alpha \in \mathbb{R}$ |
| Kısmen ayırt edilebilir durumlar | $0 \leq \lvert\langle\phi\lvert\psi\rangle\rvert \leq 1$, CBS eşitsizliği |
| Tam ayırt edilebilir kuantum durumlarının tam kümesi | Tam ortonormal vektör bazları |
| Tam ayırt edilebilir kuantum durumlarının maksimum sayısı | $\mathscr{H}$'nin kompleks boyutu |

# Saf-olmayan kuantum durumlar: Yoğunluk işlemcileri (matrisleri)

- $|u_1>, \ |u_2>, \ . \ . \ . \ . , \ |u_j>$ olası boylandırılmış durumlar

  $p_1 \ , \ p_2 \ , \ . \ . \ . \ . , \ p_j$ önsel olasılıklar

  $p_i \in [0,1] \, , \ \sum_{i=1}^{j} p_i = 1$

  $\rho = \Sigma_i p_i \, | \, u_i >< u_i |$ konveks karışım

  $Tr \, \rho = 1$ birim izlilik

  $\rho \geq 0 \iff \langle \psi | \rho | \psi \rangle \geq 0 \, , \ \forall | \psi \rangle$ pozitiflik

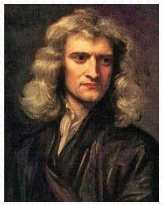
# Sistem

## Klasik Durumlar

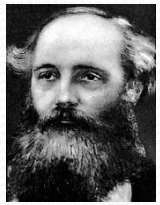
Klasik Bilişim Kuramı

Klasik Mekanik Yasaları

### Donanım Yasaları



**Newton** (1643-1727)

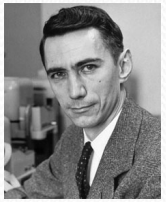


**Maxwell** (1831-1879)

### Yazılım Kuralları

(Protokoller ve Algoritmalar)

**Turing** (1912-1954)



**Shannon** (1916-2001)

## Kuantum Durumlar

Kuantum Bilişim Kuramı

Kuantum Mekaniği Yasaları

### Donanım Yasaları

### Yazılım Kuralları

(Kuantum Protokoller ve Algoritmalar)

**Bennett** (1943-…)

**Deutsch** (1953-…)

**Shor** (1959-…)

**Grover** (1961-…)

…

$|\psi\rangle \longleftrightarrow$ Saf kuantum durum

$\langle\phi|\psi\rangle = 0 \longleftrightarrow$ Tam ayırt edilebilir saf kuantum durumlar

$|\psi\rangle = \sum_i c_i |u_i\rangle, \quad c_j = \langle u_j|\psi\rangle \longleftrightarrow$ Olasılık genlikleri

$|c_j|^2 = |\langle u_j|\psi\rangle|^2 \longleftrightarrow$ Olasılıklar

## Kuantum durumlarındaki bilgi;

- koherent olarak üst-üste binmiş (süperpoze) bilgidir, } **Kuantum paralelizm**
- kopyalanamaz ve gizlidir. } **Kuantum kriptolojide koşulsuz güvenlik**
- Bileşik sistemlerin bileşenleri arasındaki kuantum korelasyonlar, (enerji gibi, momentum gibi) **yeni kuantum bilişimsel kaynaklar**dır.

| Durum | Ayrılabilir Durumlar (*Separable States*) | Dolanık Durumlar (*Entangled States*) |
|---|---|---|
| **SAF DURUMLAR** (***Pure States***) | $\lvert\Psi\rangle = \lvert\psi_1\rangle \otimes \lvert\psi_2\rangle$ <br> $\lvert\Psi\rangle \in \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ | $\lvert\Psi\rangle \neq \lvert\psi_1\rangle \otimes \lvert\psi_2\rangle$ |
| **SAF-OLMAYAN DURUMLAR** (***Mixed States***, ***Impure States***) | $\rho_{12} = \sum_{i=1}^{k} p_i \rho_1^i \otimes \rho_2^i$ <br> $p_i \in [0,1]\,, \sum_{i=1}^{k} p_i = 1$ | $\rho_{12} \neq \sum_{i=1}^{k} p_i \rho_1^i \otimes \rho_2^i$ <br> **(R. F. Werner, 1989)** |

# Kuantum dolanıklık (entanglement)

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0> \otimes |1\rangle - |1> \otimes |0\rangle)$$

- Bütün iyi biliniyor fakat parçaların durumu iyi bilinmiyor !
- Birbirlerinden çok uzak olsalar bile iki nesne tek bir yapı oluşturur.

# Tek-kübit Kuantum Mantık Geçitleri

- Hadamard geçidi (*Hadamard gate*)

$$|0\rangle \;\text{—}\boxed{\mathbf{H}}\text{—}\; \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

$$\alpha|0\rangle+\beta|1\rangle \;\text{—}\boxed{\mathbf{H}}\text{—}\; \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}}+\beta\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

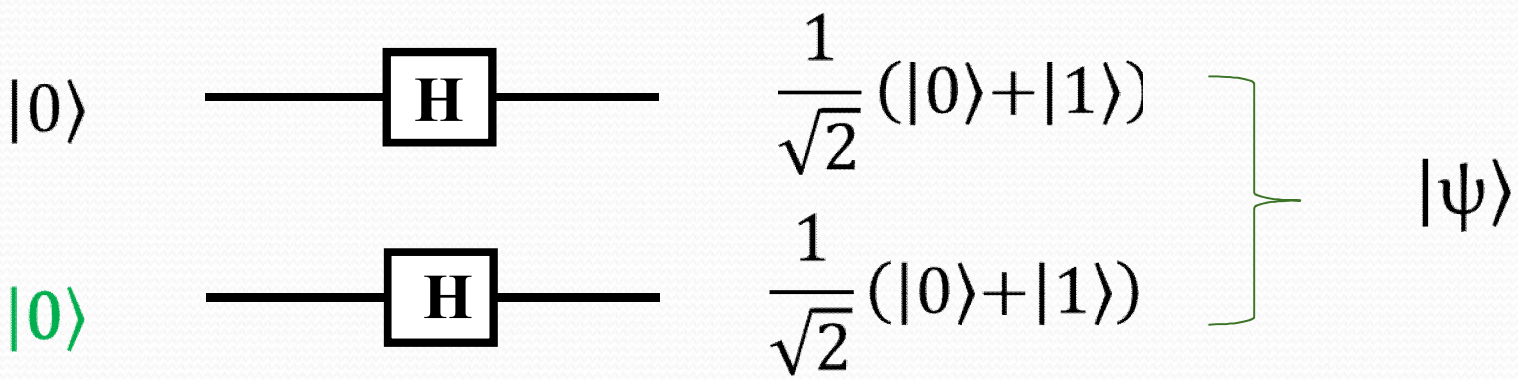
Tek **kübit** için bir baz $(\mathcal{H} \cong \mathbb{C}^2)$

$$|0\rangle=\begin{pmatrix}1\\0\end{pmatrix},\quad |1\rangle=\begin{pmatrix}0\\1\end{pmatrix}$$

$$H=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

# Paralel iki Hadamard geçidi






$$|0\rangle\otimes|0\rangle \quad \rightarrow \quad |\psi\rangle = \frac{1}{2}(|0\rangle\otimes|0\rangle+|0\rangle\otimes|1\rangle+|1\rangle\otimes|0\rangle+|1\rangle\otimes|1\rangle)$$

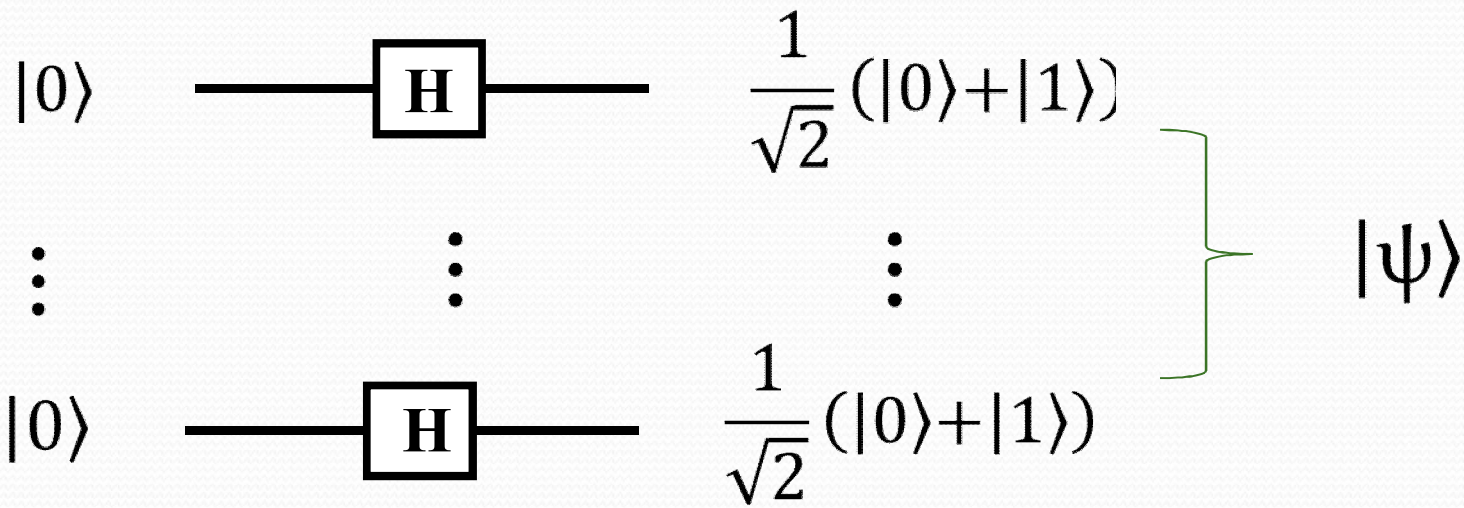
$$= \frac{1}{2}(|00\rangle+|01\rangle+|10\rangle+|11\rangle)$$

$$= \frac{1}{2}(|0\rangle+|1\rangle+|2\rangle+|3\rangle) \quad \text{(ondalık sistemde)}$$

$$= \frac{1}{2^{2/2}}\sum_{n=0}^{2^2-1}|n\rangle \quad \text{(ondalık sistemde)}$$

# Paralel n Hadamard geçidi






$$|0\rangle \otimes \cdots \otimes |0\rangle \quad \longrightarrow \quad |\psi\rangle = \frac{1}{2^{n/2}} \sum_{n=0}^{2^n-1} |n\rangle$$

(ondalık sistemde)

# Pauli geçitleri

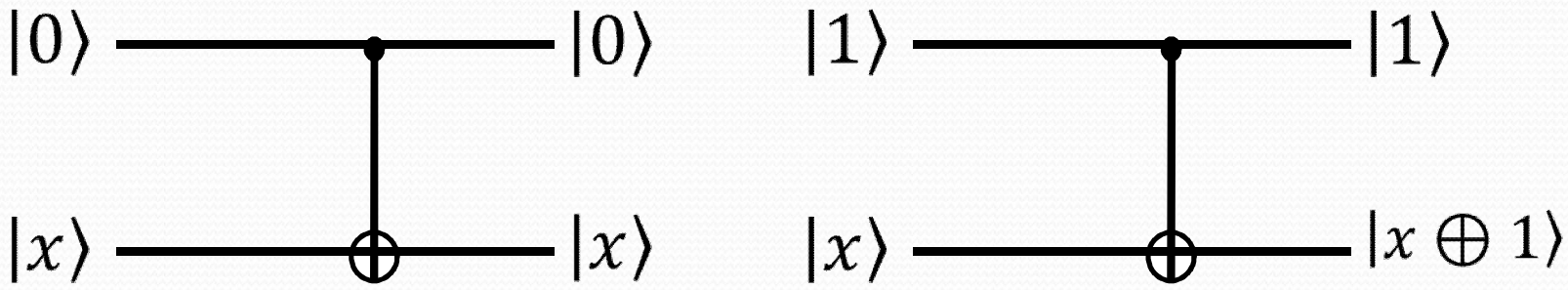
$$\alpha|0\rangle+\beta|1\rangle \;\text{—}\boxed{\mathbf{X}}\text{—}\; \alpha|1\rangle+\beta|0\rangle$$

$$\alpha|0\rangle+\beta|1\rangle \;\text{—}\boxed{\mathbf{Y}}\text{—}\; i\left(\alpha|1\rangle-\beta|0\rangle\right)$$

$$\alpha|0\rangle+\beta|1\rangle \;\text{—}\boxed{\mathbf{Z}}\text{—}\; \alpha|0\rangle-\beta|1\rangle$$

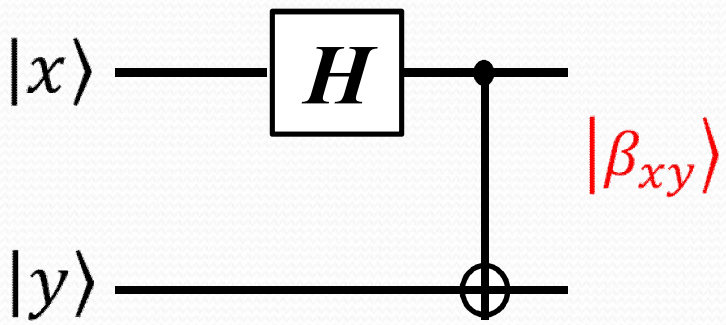
# İki-kübit Geçitleri

**CNOT (Kontrollü Değilleme) geçidi**



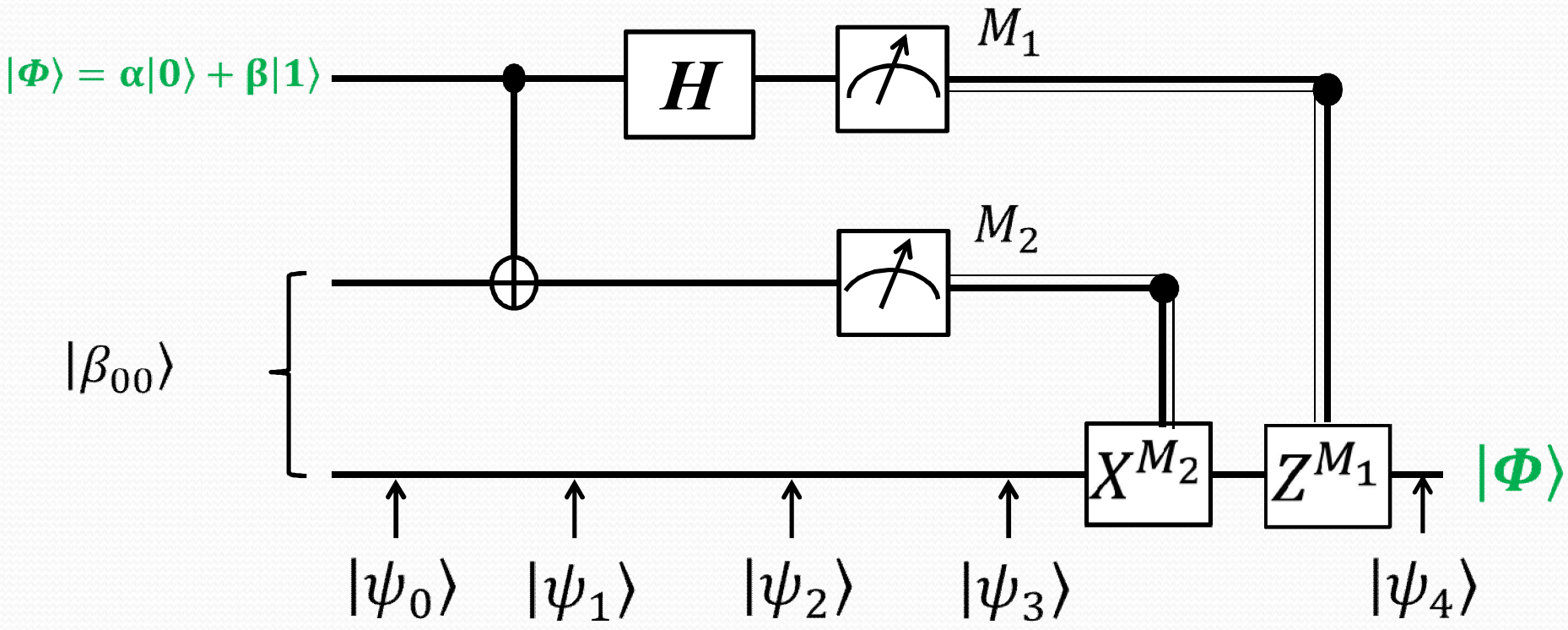
# Dolanıklaştırıcı kuantum devresi:

## Bell (EPR) durumları

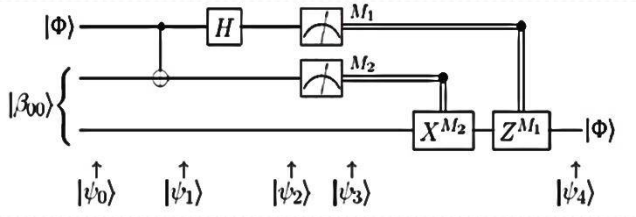



| $\|x\rangle$ | $\|y\rangle$ | Çıkış |
|---|---|---|
| $\|0\rangle$ | $\|0\rangle$ | $\frac{1}{\sqrt{2}}(\|00\rangle+\|11\rangle) \equiv \|\beta_{00}\rangle$ |
| $\|0\rangle$ | $\|1\rangle$ | $\frac{1}{\sqrt{2}}(\|01\rangle+\|10\rangle) \equiv \|\beta_{01}\rangle$ |
| $\|1\rangle$ | $\|0\rangle$ | $\frac{1}{\sqrt{2}}(\|00\rangle-\|11\rangle) \equiv \|\beta_{10}\rangle$ |
| $\|1\rangle$ | $\|1\rangle$ | $\frac{1}{\sqrt{2}}(\|01\rangle-\|10\rangle) \equiv \|\beta_{11}\rangle$ |

# Kuantum **Uz-aktarım** (***Teleportation***)



# Kuantum Uz-aktarım




- $$|\psi_0\rangle = |\boldsymbol{\Phi}\rangle \otimes |\beta_{00}\rangle$$
$$= (\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)]$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)]$$

$$|\psi_2\rangle = \frac{1}{2}[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)]$$
$$= \frac{1}{2}[|00\rangle(\boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle) + |01\rangle(\boldsymbol{\alpha}|\mathbf{1}\rangle + \boldsymbol{\beta}|\mathbf{0}\rangle)$$
$$+|10\rangle(\boldsymbol{\alpha}|\mathbf{0}\rangle - \boldsymbol{\beta}|\mathbf{1}\rangle) + |11\rangle(\boldsymbol{\alpha}|\mathbf{1}\rangle - \boldsymbol{\beta}|\mathbf{0}\rangle)]$$

***(Bennett, Brassard, Crepeau, Jozsa, Peres and Wootters, 1993)***

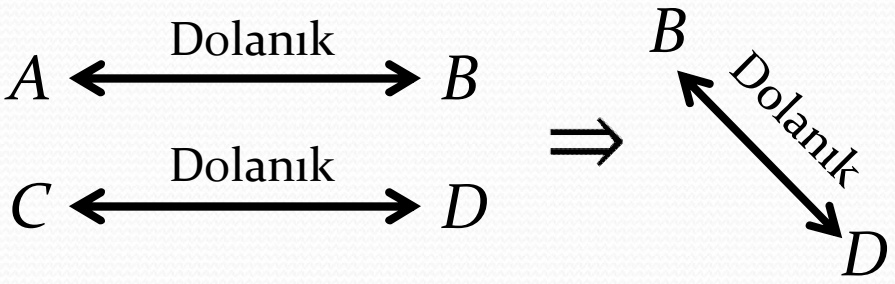
# Bir kübitin uz-aktarımı için

Fatura:

1 ebit+2 bit =??? ₺

# Diğer öncü kuantum bilişim uygulamaları

- Dolanıklık trampası (dolanıklığın uz-aktarımı)




- Kuantum yoğun kodlama (dolanıklıktan yararlanarak kuantum durumlarıyla klasik bilgi iletme kapasitelerinin ikiye katlanması)
- **Shor algoritmasıyla büyük pozitif tamsayıların kısa sürede asal çarpanlarına ayrılması**
- Kuantum sistemlerin simülasyonları
- Kuantum kriptoloji
- Klasik olarak gerçekleştirilebilir pek çok süreç ve yükümlülükte hızlanırlık

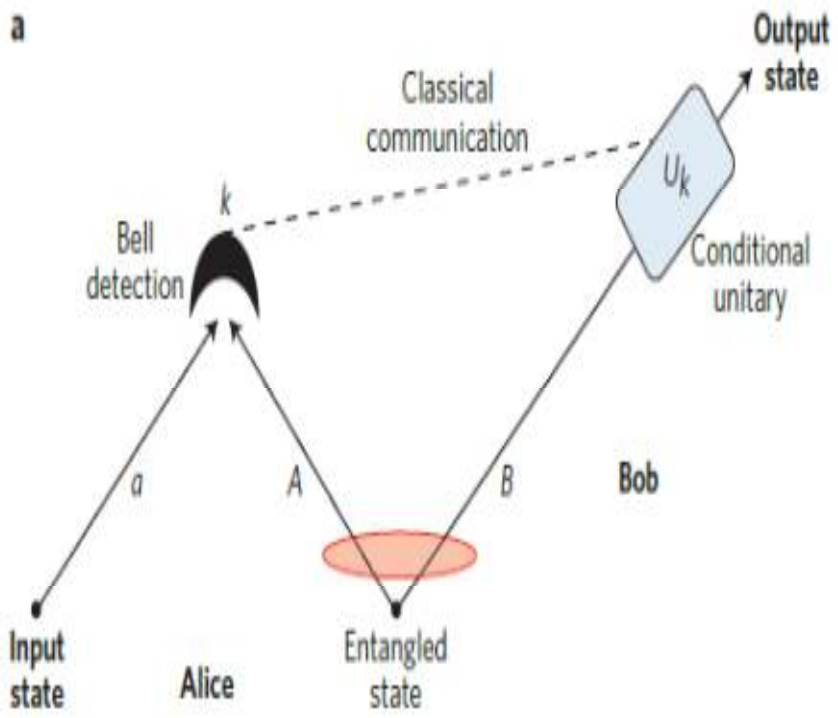
nature photonics

REVIEW ARTICLE

PUBLISHED ONLINE: 29 SEPTEMBER 2015 | DOI: 10.1038/NPHOTON.2015.154

# Advances in quantum teleportation

S. Pirandola[1*], J. Eisert[2], C. Weedbrook[3], A. Furusawa[4] and S. L. Braunstein[1]




Basic protocol. Alice's unknown input state is teleported to Bob using a shared entangled state and a classical communication channel. Alice performs a Bell state measurement on her systems, *a* and *A*, and classically communicates the outcome to Bob. Using the measurement result, Bob applies the conditional unitary *U* to his system *B*, thus retrieving an output state that is an exact replica of Alice's input state in the ideal case ($F = 1$).

nature photonics

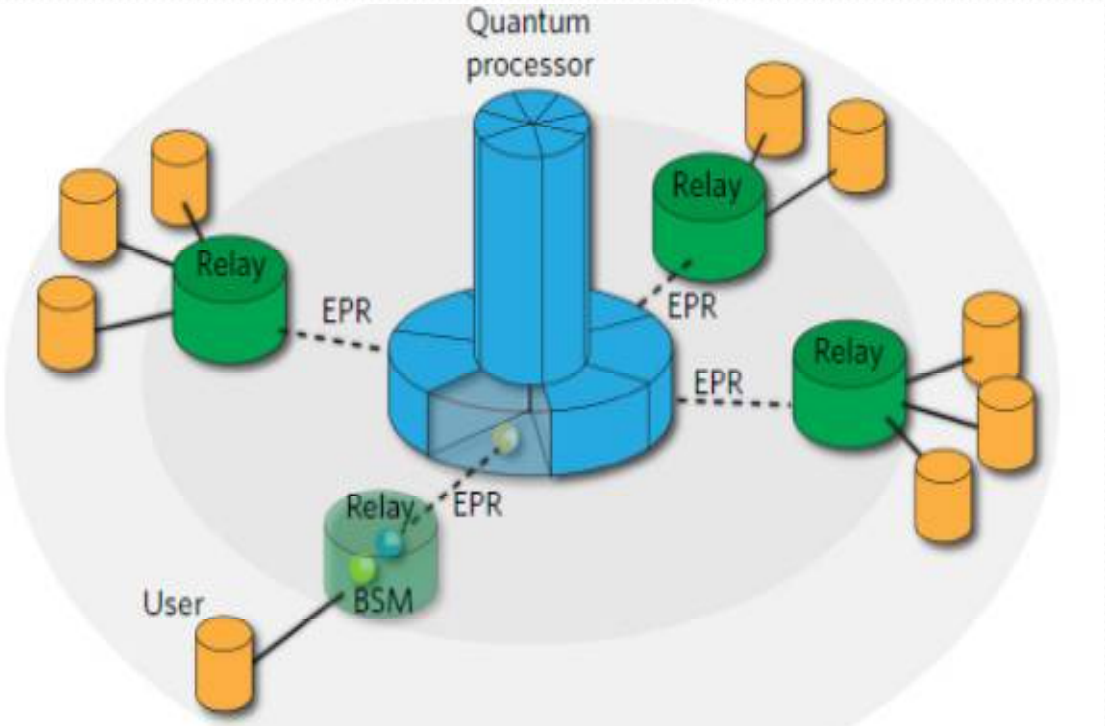
LETTERS

PUBLISHED ONLINE: 19 SEPTEMBER 2016 | DOI: 10.1038/NPHOTON.2016.179

# Quantum teleportation with independent sources and prior entanglement distribution over a network

Qi-Chao Sun[1,2,3], Ya-Li Mao[1,2], Si-Jing Chen[4], Wei Zhang[5], Yang-Fan Jiang[1,2], Yan-Bao Zhang[6], Wei-Jun Zhang[4], Shigehito Miki[7], Taro Yamashita[7], Hirotaka Terai[7], Xiao Jiang[1,2], Teng-Yun Chen[1,2], Li-Xing You[4], Xian-Feng Chen[3], Zhen Wang[4], Jing-Yun Fan[1,2], Qiang Zhang[1,2]* and Jian-Wei Pan[1,2]*

Schematics of a quantum network with a star-topology structure.




(BSM, Bell-state measurement)

The central node hosts a quantum processor and shares entanglement with the relay nodes. The end users access the central quantum processor by teleporting the quantum states to the central node via the relay nodes. The relay nodes perform a BSM, and feedforward the measurement outcomes to the central processor.

NATURE | VOL 390 | 11 DECEMBER 1997



# Experimental quantum teleportation

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**Quantum teleportation–the transmission and reconstruction over arbitrary distances of the state of a quantum system–is demonstrated experimentally. During teleportation, an initial photon which carries the polarization that is to be transferred and one of a pair of entangled photons are subjected to a measurement such that the second photon of the entangled pair acquires the polarization of the initial photon. This latter photon can be arbitrarily far away from the initial one. Quantum teleportation will be a critical ingredient for quantum computation networks.**

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# Quantum teleportation over 143 kilometres using active feed-forward

Xiao-Song Ma[1,2]†, Thomas Herbst[1,2], Thomas Scheidl[1], Daqing Wang[1], Sebastian Kropatschek[1], William Naylor[1], Bernhard Wittmann[1,2], Alexandra Mech[1,2], Johannes Kofler[1,3], Elena Anisimova[4], Vadim Makarov[4], Thomas Jennewein[1,4], Rupert Ursin[1] & Anton Zeilinger[1,2]

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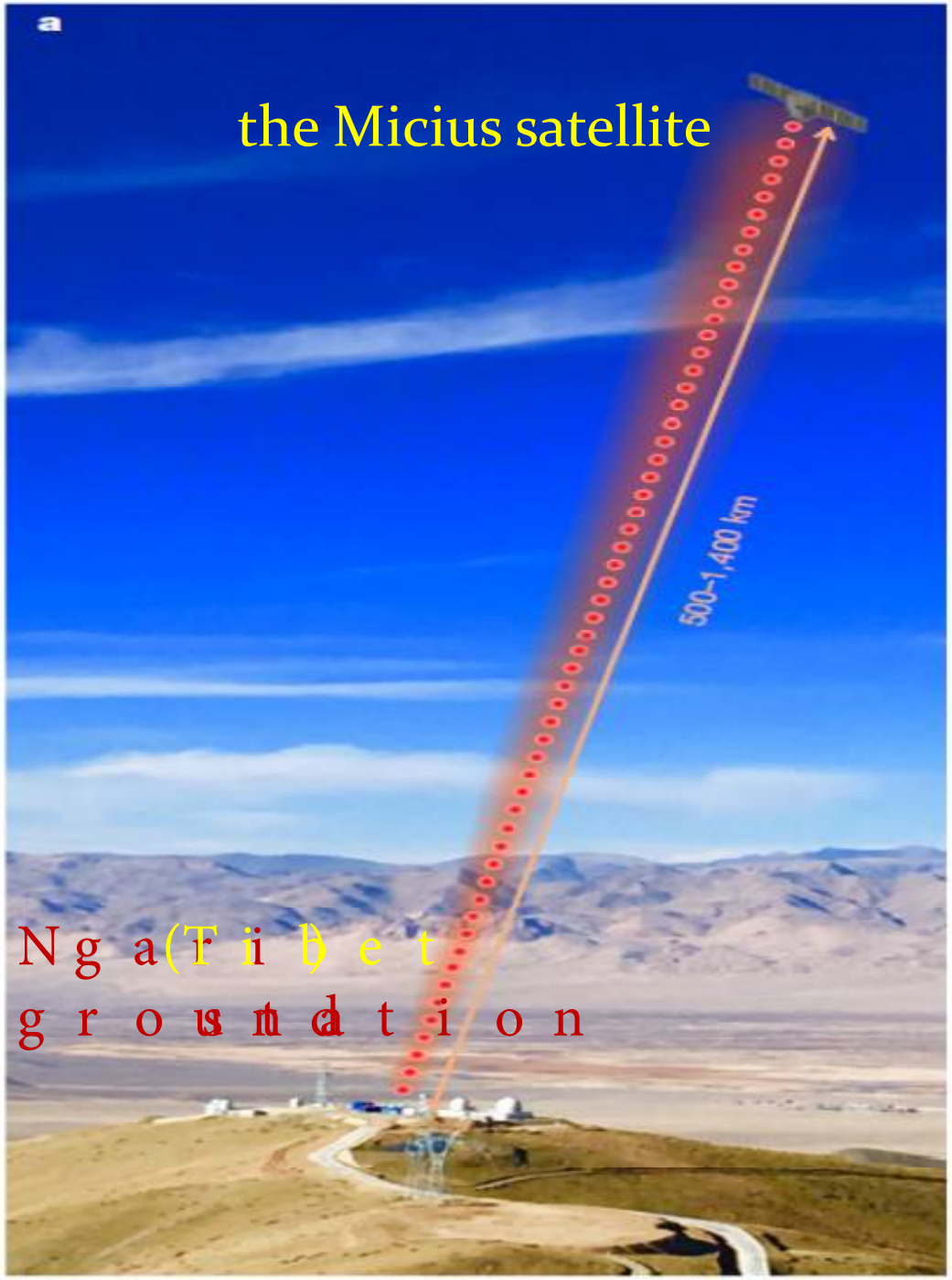
# Ground-to-satellite quantum teleportation

- Ji-Gang Ren1,2, Ping Xu1,2, Hai-Lin Yong1,2, Liang Zhang2,3, Sheng-Kai Liao1,2, Juan Yin1,2, Wei-Yue Liu1,2, Wen-Qi Cai1,2,Meng Yang1,2, Li Li1,2, Kui-Xing Yang1,2, Xuan Han1,2, Yong-Qiang Yao4, Ji Li5, Hai-Yan Wu5, Song Wan6, Lei Liu6,Ding-Quan Liu3, Yao-Wu Kuang3, Zhi-Ping He3, Peng Shang1,2, Cheng Guo1,2, Ru-Hua Zheng7, Kai Tian8, Zhen-Cai Zhu6,Nai-Le Liu1,2, Chao-Yang Lu1,2, Rong Shu2,3, Yu-Ao Chen1,2, Cheng-Zhi Peng1,2, Jian-Yu Wang2,3 & Jian-Wei Pan1,2

**An arbitrary unknown quantum state cannot be measured precisely or replicated perfectly. However, quantum teleportation enables unknown quantum states to be transferred reliably from one object to another over long distances, without physical travelling of the object itself.**




**Overview of the ground-to-satellite quantum teleportation of a photon over distances up to 1,400 km**

**Our demonstration of a ground-to-satellite uplink for reliable and ultra-long-distance quantum teleportation is an essential step towards a global-scale quantum internet.**

A schematic of the satellite overhead photographs Ngari ground station. The separation between the ground station and the satellite is about 500 km to 1,400 km for quantum teleportation

***Değerli vakitleriniz, sabrınız ve sorularınız için,***

***Sayın Bilsay KURUÇ'a bu konuşmayı yapmak üzere davet ettiği için,***

***Sayın Orhan BURSALI'ya bu oturumu yönettiği için,***

***D. Duran ve S. Yılmaz'a bu sunumu hazırlamamdaki yardımları için,***

***TEŞEKKÜRLER...***